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Overview of Theoretical Prospects for Understanding the Values of Fundamental Constants [and Discussion]

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Overview of theoretical prospects for understanding the values of fundamental constants

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This is a brief summary of the talk given at the Meeting.

The membership of a list of 'fundamental' constants necessarily depends on who is compiling the list. A hydrodynamicist might reasonably include the density and viscosity of water, while an atomic physicist would doubtless include the proton mass and electronic charge. This talk deals with a different sort of list: a list of the constants that appear in the laws of nature at the deepest level that we yet understand, constants whose value we cannot calculate with precision in terms of more fundamental constants, not just because the calculation is too complicated (as for the viscosity of water or the mass of the proton) but because we do not know of anything more fundamental. The membership of such a list of fundamental constants thus reflects our present understanding of fundamental physics. Also, each constant on the list is a challenge for future work, to try to explain its value.

The parameters that appear at the most fundamental level in our present theories of elementary particles are (1) the electroweak and strong gauge couplings g_1, g_2, g_3, and (2) the masses and self couplings of the 'Higgs' scalars, and (3) the coupling constants for the interaction of the scalars to quarks and leptons. The gauge couplings themselves determine† the observables $e^2 = g_1^2 g_2^2/(g_1 + g_2^2)$ and $\sin^2\theta = g_2^2/(g_1^2 + g_2^2)$, which are experimentally known to be $4\pi/137$ and 0.22. The scalar masses and self-couplings determine the scalar vacuum expectation values, and hence the Fermi coupling $G_F = \langle\phi^0\rangle^{-2}/\sqrt{2}$, which is experimentally known to be $(293\,\mathrm{GeV})^{-2}$. Combined with the gauge couplings, these vacuum expectation values also determine the W and Z masses, with predicted values of about 83 and 93 GeV, which are now happily in agreement with experiment. Finally, the scalar vacuum expectation values and scalar–fermion couplings determine the quark and lepton masses, which experimentally range over more than three orders of magnitude. From these experimental masses we deduce that the scalar–fermion coupling constants are much smaller than the gauge couplings and very different from each other, but we have no clear idea of why this should be so.

It is somewhat misleading to list the gauge couplings as fundamental parameters. We know in the case of quantum chromodynamics that the QCD coupling is not a constant; rather g_3^2 varies with the energy E as $24\pi^2/25 \ln(E/\Lambda_3)$, (for E below the bottom mass). The constant Λ_3 determines the general scale of strong interaction physics, including the current-algebra constant F_π and the 'constituent quark' mass m_q. Experimentally $\Lambda_3 \sim 150\,\mathrm{MeV}$, $F_\pi \sim 190\,\mathrm{MeV}$, and $m_q \approx \frac{1}{3} m_N = 310\,\mathrm{MeV}$. We do not yet know how to calculate F_π and m_q from Λ_3. The only place

† When I say that one set of constants 'determines' a second set, I mean that we think that the first set is more fundamental, and that the values of the second set are what they are because of the values taken by the first set, in the way for instance that the energy levels of atoms are what they are because of the values taken by the electron's mass and charge and Planck's constant. I do not mean that we have historically deduced the values of the second set from those of the first set: in fact the opposite is more likely to be the case.

in low energy strong interaction physics where another scale factor intrudes is in the pion mass, which is given in terms of the 'bare quark' mass $m_{q'}$ (the one determined by the scalar couplings) by $m_\pi^2 \approx \Lambda_3 m_{q'}$; it is from this relation that we deduce the bare u and d quark mass scale of a few MeV.

It is possible that the scalars are not really elementary, but bound by some sort of extra strong or 'technicolour' force, with a scale parameter Λ_4 very different from Λ_3. In this case it is Λ_4 that determines the Fermi coupling, from whose value we deduce that Λ_4 must be about 300 GeV. The problem with this view is that it is difficult to see where the quark and lepton masses come from. One possibility is that there is yet another extra extra strong force, 'extended technicolour', with a scale factor $\Lambda_5 \gg \Lambda_4$, which connects technifermions of mass *ca.* Λ_4 with the otherwise massless quarks and leptons. In this case the quark and lepton masses are theoretically determined to be of order Λ_4^3/Λ_5^2, from which we infer that Λ_5 is in the multi-TeV range.

One other observable parameter must be mentioned, the Newton constant of gravitation G. An increasingly popular view (which I share) is that this is not a fundamental constant, but related to an energy scale M_G at which some entirely new physics enters. On this view, the effective Lagrangian that describes gravitation at ordinary energies is a power series

$$\mathscr{L}_{\text{eff}} = \sqrt{g}[c_0 M_G^4 + c_1 M_G^2 R + c_2 R^2 + c_2' R^{\mu\nu} R_{\mu\nu} + c_3 M_G^{-2} R^3 + \ldots],$$

with dimensionless constants c_i. If $c_1, c_2, c_2', c_3, \ldots$ are of order unity, then the only terms we would have had any chance of observing experimentally are the c_0 and c_1 terms, leaving us with conventional general relativity plus a cosmological constant. If we define M_G so that $c_1 \equiv 1$, then from the experimental value of G we conclude that $M_G = (16\pi G)^{-\frac{1}{2}} = 1.72 \times 10^{18}$ GeV. Unfortunately c_0 turns out not to be quite of order unity; from upper limits on the cosmological constant, we conclude that $c_0 < 10^{-119}$. No one knows why.

Not only does the QCD (and technicolour and extended technicolour) coupling vary with energy: the same is true of the electroweak couplings g_1 and g_2, but with a slower rate of variation. When g_1, g_2, and g_3 are extrapolated to very high energy, they are found to come together (with relative normalizations fixed by the menu of quarks and lepton quantum numbers) at an energy of order 10^{15} GeV. This allows one plausibly to suppose that the strong and electroweak gauge groups are subgroups of some 'grand unified' gauge group. On this view, it is the symmetry breaking scale M_{GUT} that determines the point where g_1, g_2, and g_3 come together, so M_{GUT} must be of order 10^{15} GeV, and it is the GUT coupling g_{GUT} that provides the common value of g_1, g_2, and g_3 at this energy. The fact that Λ_1, Λ_2, and Λ_3 are enormously different from each other and from M_{GUT} is then simply explained by the smallness of g_{GUT}. Unfortunately, in the case of electroweak symmetry breaking by elementary scalars, it is a mystery why the vacuum expectation value $\langle\phi\rangle \approx 300$ GeV is so much less than M_{GUT}. This is the well known hierarchy problem. Another problem that is not so often mentioned is why the renormalization scale Λ_{GUT} of the grand gauge group is so different from M_{GUT}, or in other words, why is g_{GUT}^2 so small?

The only theories I know that provide much hope of being able to calculate gauge couplings like g_{GUT} from fundamental principles are those that derive the gauge couplings from gravity in 6 or more dimensions. An extended version of the portion of this talk that dealt with this topic will be published in the proceedings of the Fourth Workshop on Grand Unification and the Shelter Island II Conference, so I will not summarize it here.

Gravitation does not seem to admit a simple quantum-mechanical description, whether in four or more dimensions. As already mentioned, it seems likely that really new physics enters at

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the Planck scale of 1.7×10^{18} GeV, and this will involve new dimensionless ratios of coupling parameters. I think it is likely that all these apparently fundamental constants will ultimately be determined by a condition of consistency of quantum mechanics with relativity, a condition that requires that all couplings have values that place the theory on a trajectory that is attracted to an ultraviolet fixed point.

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Discussion

H. B. Nielsen (*Bohr Institute, Copenhagen, Denmark*). Professor Weinberg seems to cling rather strongly to wanting a continuum theory rather than one with a fundamental cut-off (a lattice or whatever), since he even wants coupling parameters to be adjusted to an ultraviolet fixed point to make the continuum work. Would Professor Weinberg not believe a fundamental cut-off theory?

S. Weinberg. I cannot say whether I would believe a fundamental cut-off theory until I see one. I shall just say that I hope that whatever cut-off is provided by nature will not work too well. It would be a great pity if such a cut-off would allow us to formulate theories with arbitrary coupling parameters, because then we would then have no idea of what it is that determines these parameters, apart perhaps for ideas like the anthropic principle. The great thing about the requirement of attraction to an ultraviolet fixed point is that in principle it can determine all or all but a finite number of coupling parameters.

J. G. Taylor (*Department of Mathematics, King's College London, Strand, London WC2R 2LS, U.K.*). After this most exciting idea of Professor Weinberg's to calculate the gauge coupling constant in higher-dimensional theories there is still the problem of ultraviolet divergences at higher loops hanging over it all, especially those arising from the gravitational radiative corrections. It is known that higher dimensional theories have far worse ultraviolet divergences than lower dimensional ones. Thus unextended supersymmetric Yang–Mills theories have infinite S-matrix elements even at 1 loop (Green & Schwarz 1982; Ragiadakos & Taylor 1983), whereas recently the dimensionally reduced theory in 4 dimensions, $N = 4$ super-Yang–Mills, is finite to all orders (Mandelstam 1982; Howe *et al.* 1982). It may then be dangerous to assume that the quantum fluctuations in higher-dimensional theories will allow neglect of higher-loop orders in some situations even if such neglect may be ultimately justified in a strictly four-dimensional theory. It may also be incorrect to assume that renormalization-group arguments, now restricted to a finite-dimensional subset of the parameter space, will be satisfactory in the higher (space–time) dimensional theories considered by Professor Weinberg.

S. Weinberg. In the work on higher-dimensional theories discussed here, the neglect of higher loops was justified as a consequence of the presence of a large number of matter fields. This justification is of course not rigorous, and in any case may not apply in the real world. However, I do not think that the problem is any worse in six or more dimensions than in four. A non-renormalizable theory is just a field theory with an infinite number of coupling parameters, all those needed to provide counterterms to the ultraviolet divergences. The neglect of higher loops is justified in such a theory with large numbers of matter fields as long as we assume that the higher coupling parameters are not much larger than would be generated by radiative corrections. Also, the renormalization group approach applies here as it does in renormalizable theories. But at any rate, we do not yet have a four-dimensional renormalizable theory of gravitation, nor in my view are we likely ever to have one.

References

Green, M. & Schwarz, J. 1982 In *Proceedings of the International High Energy Physics Conference*, *Paris*. Geneva: CERN.
Howe, P., Stelle, K. S. & Townsend, P. 1982 In *Proceedings of the International High Energy Physics Conference*, *Paris*. Geneva: CERN.
Mandelstam, S. 1982 In *Proceedings of the International High Energy Physics Conference*, *Paris*. Geneva: CERN.
Ragiadakos, C. & Taylor, J. G. 1983 *Phys. Lett.* B (In the press.)

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